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Research article

Fundamental Groups and Fractal Retractions of Chaotic Narlikar and Karmarkar Space

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Abstract

In this article we will introduce a new type of fundamental group in chaotic Narlikar and Karmarkar space. The fundamental group of geodesics for chaotic Narlikar and Karmarkar space will be deduced. The fundamental group of the end of limit folding in chaotic Narlikar and Karmarkar space will be presented. The connection between the fundamental group and the fractal retractions are discussed. The relation between the fundamental group and the fractal foldings are discussed.

Keywords: Chaotic Narlikar and Karmarkar space, Retraction, Folding, Fundamental group.

Mathematical subject classification: 53A35, 51H05, 58C05.

Introduction and definitions

Narlikar and Karmarkar space represents one of the most intriguing and emblematic discoveries in the history of geometry. Although if it were introduced for a purely geometrical purpose, they came into prominence in many branches of mathematics and physics. This association with applied science and geometry generated synergistic effect: applied science gave relevance to Narlikar and Karmarkar space and Narlikar and Karmarkar space allowed formalizing practical problems [25, 26].

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In vector spaces and linear maps; topological spaces and continuous maps; groups and homomorphisms together with the distinguished family of maps is referred to as a category. An operator which assigns to every object in one category a corresponding object in another category and to every map in the first a map in the second in such a way that compositions are preserved and the identity map is taken to the identity map is called a functor. Thus, we may summarize our activities thus far by saying that we have constructed a functor (the fundamental group functor) from the category of pointed spaces and maps to the category of groups and homomorphisms. Such functors are the vehicles by which one translates topological problems into algebraic problem [10, 12, 14, 27, 37-40].

Most folding problems are attractive from a pure mathematical standpoint, for the beauty of the problems themselves. The folding problems have close connections to important industrial applications. Linkage folding has applications in robotics and hydraulic tube bending. Paper folding has application in sheet-metal bending, packaging, and air –bag folding [11]. Also, used folding to solve difficult problems related to shell structures in civil engineering and aero space design, namely buckling instability [11]. Isometric folding between two Riemannian manifold may be characterized as maps that send piecewise geodesic segments to a piecewise geodesic segments of the same length [7, 8, 21]. For a topological folding the maps do not preserves lengths[19, 35], i.e. A map $\Im : M \to N$, where M and N are c^{∞}–Riemannian manifolds of dimension m and n respectively is said to be an isometric folding of M into N , iff for any piecewise geodesic path $\gamma : J \to M$, the induced path $\Im \circ \gamma : J \to N$ is a piecewise geodesic and of the same length as γ . If $\Im$ does not preserve length, then $\Im$ is a topological folding [15, 17, 34, 36].

An n-dimensional topological manifold M is a Hausdorff topological space with a countable basis for the topology which is locally homeomorphic to R^n. If $h : U \to U'$ is a homeomorphism of $U \subseteq M$ onto $U' \subseteq R^n$,then h is called a chart of M and U is the associated chart domain. A collection (h_α, u_α) is said to be an atlas for M if $\bigcup_{\alpha \in A} U_\alpha = M$. Given two charts h_α, h_β such that $U_{\alpha\beta} = U_\alpha \bigcap U_\beta \neq \emptyset$,the transformation chart $h_\beta \circ h_\alpha^{-1}$ between open sets of R^n is defined, and if all of these charts transformation are c^{∞} -mappings, then the manifolds under consideration is a c^{∞} - manifolds. A differentiable structure on M is a differentiable atlas and a differentiable manifold is a topological manifold with a differentiable structure [41-44]. M may have other structures as colors, density or any physical structures. The number of structures may be infinite. In this case the manifold is said to be a chaotic manifold and may become relevant to vacuum fluctuation and chaotic quantum field theories. The magnetic field of a magnet bar is a kind of chaotic 1-dimensional manifold represented by the magnetic flux lines. The geometric manifold is the magnetic bar itself [1] .

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Fuzzy manifolds are special type of the category of chaotic manifolds [2-5]. Usually we denote by M = $M_{012..h}$ to a chaotic manifolds, where M_{0h} is the geometric (essential) manifold and the associated pure chaotic manifolds, the manifolds with physical characters, are denoted by $M_{1h},...,M_{\infty h}$ [16, 24, 28, 29, 31, 32].

A subset A of a topological space X is called a retract of X if there exists a continuous map $r: X \to A$ such that $r(a)=a, \forall a \in A$ where A is closed and X is open [9, 13, 20, 33]. Also, let X be a space and A a subspace. A map $r: X \to A$ such that $r(a)=a$, for all $a \in A$, is called a retraction of X onto A and A is the called a retract of X .This can be re stated as follows. If $i: X \to A$ is the inclusion map, then $r: X \to A$ is a map such that $ri = id_a$. If, in addition, $ri \approx id_X$, we call r a deformation retract and A a deformation retract of X [6] . Another simple-but extremely useful-idea is that of a retract. If $A, X \subset M$, then A is a retract of X if there is a commutative diagram [18, 22, 23, 30].

$$\begin{tikzcd} A \arrow[r, "i"'] \arrow[rr, bend left, "id_A"] & X \arrow[r, "r"'] & A \end{tikzcd}$$

Main results

Theorem 1. The fundamental groups for geodesics of the chaotic Narlikar and Karmarkar space K_{ih} by using Lagrangian equations are isomorphic to identity groups.

Proof. The chaotic Narlikar and Karmarkar space K_{ih} is defined whose metric in the form

$$ds^2 = dx_1^2 - \upsilon^p dx_2^2 - \upsilon^q dx_3^2 - \upsilon^r dx_4^2 \qquad ...(1)$$

where $\upsilon = 1 + kx_1$, k is a function of time t, and p,q are constant connected by the relations $p+q+r=2$, $pq+qr+rp=0$.

Then the coordinates of chaotic Narlikar and Karmarkar space take the form:

$$\left.\begin{aligned} x &= x_1 + c_1 \\ y &= i\upsilon^{p/2} x_2 + c_2 \\ z &= i\upsilon^{q/2} x_3 + c_3 \\ \tau &= i\upsilon^{r/2} x_4 + c_4 \end{aligned}\right\} \qquad ...(2)$$

Where c_1, c_2, c_3 and c_4 are the constant of integration.

Now, by using the Lagrangian equations $\frac{d}{ds}\left(\frac{\partial T}{\partial x'_i}\right)-\frac{\partial T}{\partial x_i}=0 \quad ,i=1,2,3,4,5$.

To deduce a chaotic geodesic which is a subset of chaotic Narlikar and Karmarkar space K_{ih}, since $T=\frac{1}{2}ds^{-2}$, this yields

$$T=\frac{1}{2}\left\{x_1^{\backslash 2}-\upsilon^p x_2^{\backslash 2}-\upsilon^q x_3^{\backslash 2}-\upsilon^r x_4^{\backslash 2}\right\} \quad (3)$$

Then, the Lagrangian equations are:

$$\frac{d}{ds}\left(x_1^{\backslash}\right)-\frac{1}{4}\left(pk\upsilon^{\frac{p}{2}-1}x_2^{\backslash 2}-qk\upsilon^{\frac{q}{2}-1}x_3^{\backslash 2}-rk\upsilon^{\frac{r}{2}-1}x_4^{\backslash 2}\right)=0 \quad (4)$$

$$\frac{d}{ds}\left(\upsilon^p x_2^{\backslash}\right)=0 \quad (5)$$

$$\frac{d}{ds}\left(\upsilon^q x_3^{\backslash}\right)=0 \quad (6)$$

$$\frac{d}{ds}\left(\upsilon^r x_4^{\backslash}\right)=0 \quad (7)$$

$$x_1k_1\left(p\upsilon^{\frac{p}{2}-1}x_2^{\backslash 2}+q\,p\upsilon^{\frac{q}{2}-1}x_3^{\backslash 2}+r\,\upsilon^{\frac{r}{2}-1}x_4^{\backslash 2}\right)=0 \quad (8)$$

From equation (7), we obtain $\upsilon^r x_4^{\backslash}=$ constant, say β, if $\beta=0$, we have two cases

(i) $x_4^{\backslash}=0$, so $x_4=$ constant $=\alpha_1$, if $\alpha_1=0$. Then, we obtain the following coordinate

$$\left.\begin{aligned} x&=x_1+c_1\\ y&=i\upsilon^{\frac{p}{2}}x_2+c_2\\ Z&=i\upsilon^{\frac{q}{2}}x_3+c_2\\ \tau&=c_4\end{aligned}\right\} \quad ...(9)$$

which is a geodesic in hyper affine subspace K_{1ih} of chaotic Narlikar and Karmarkar space. Also, these geodesic is a retraction of Narlikar and Karmarkar space. Again $\pi_1\{K_{ih}-(\mu_i)\}\approx\pi_1(K_{1ih})$ and therefore $\pi_1(K_{1_{ih}})$ is isomorphic to identity group.

(ii) If $\upsilon=0$, then we obtain the following geodesic

$$\left.\begin{aligned} x&=x_1+c_1\\ y&=c_2\\ Z&=c_3\\ \tau&=c_4\end{aligned}\right\} \quad ...(10)$$

which is affine subspace K_{2ih} in chaotic Narlikar and Karmarkar space which is a retraction. Also, $\pi_1\{K_{ih}-(\mu_i)\}\approx\pi_1(K_{2ih})$ and therefore $\pi_1(K_{2ih})$ is isomorphic to identity group.

From equation (5), we have $\upsilon^{p}x_2^{\backslash}=$ constant, say β_2, if $\beta_2=0$, then we have two cases:

(a) $x_2^{\backslash}=0$, so, $x_2=$ constant, say α_2, if $\alpha_2=0$. Then, we get the following geodesic in hyper affine subspace K_{3ih} of chaotic Narlikar and Karmarkar space which is a retraction.

$$\left.\begin{aligned} x&=x_1+c_1\\ y&=c_2\\ Z&=i\upsilon^{\frac{q}{2}}x_3+c_3\\ \tau&=i\upsilon^{\frac{r}{2}}x_4+c_4 \end{aligned}\right\} \qquad ...(11)$$

Also, $\pi_1\{K_{ih}-(\mu_i)\}\approx\pi_1(K_{3ih})$ and therefore $\pi_1(K_{3ih})$ is isomorphic to identity group .

(b) If $\upsilon=0$, then we get the retraction as in equation (10) and the fundamental group $\pi_1(K_{2ih})$ is isomorphic to identity group .

From equation (6), we get $\upsilon^{q}x_3^{\backslash}=$ constant; say β_3, if $\beta_3=0$, then we have two cases

(c) $x_3^{\backslash}=0$, so, $x_3=$ constant, say α_3, if $\alpha_3=0$. Then, we have the following retraction in hyper affine subspace K_{4ih} of chaotic Narlikar and Karmarkar space which is a geodesic

$$\left.\begin{aligned} x&=x_1+c_1\\ y&=i\upsilon^{\frac{p}{2}}x_2+c_2\\ Z&=c_3\\ \tau&=i\upsilon^{\frac{r}{2}}x_4+c_4 \end{aligned}\right\} \qquad ...(12)$$

Also, $\pi_1\{K_{ih}-(\mu_i)\}\approx\pi_1(K_{4ih})$ and therefore $\pi_1(K_{4ih})$ is isomorphic to identity group.

(d) If $\upsilon=0$, then we obtain the geodesic as in equation (10) and also the fundamental group $\pi_1(K_{2ih})$ is isomorphic to identity group .

From equation (8), we get either $x_1k'=0$ or $p\upsilon^{p-1}x_2^{\backslash 2}+q\,\upsilon^{q-1}x_3^{\backslash 2}+r\,\upsilon^{\gamma-1}x_4^{\backslash 2}=0$. Then we have either $x_1=0$, and we have the following retraction in hyper affine subspace K_{5ih} of chaotic Narlikar and Karmarkar space which is a retraction. Again the coordinate is

$$\left.\begin{aligned} x &= c_1 \\ y &= i\upsilon^{\frac{p}{2}} x_2 + c_2 \\ Z &= i\upsilon^{q} x_3 + c_3 \\ \tau &= i\upsilon^{r} x_4 + c_4 \end{aligned}\right\} \qquad ...(13)$$

Also, $\pi_1\{K_{ih} - (\mu_i)\} \approx \pi_1(K_{5ih})$ and therefore the fundamental group $\pi_1(K_{5ih})$ is isomorphic to identity group.

Again or x_2, x_3, x_4 are constant, say $x_2 = a_1, x_3 = a_2, x_4 = a_3$ and we have the following retraction in hyper affine subspace K_{6ih} of chaotic Narlikar and Karmarkar space which is a retraction. Again the coordinate is

$$\left.\begin{aligned} x &= x_1 + c_1 \\ y &= ia_1\upsilon^{\frac{p}{2}} + c_2 \\ Z &= ia_2\upsilon^{\frac{q}{2}} + c_3 \\ \tau &= ia_3\upsilon^{\frac{r}{2}} + c_4 \end{aligned}\right\} \qquad ...(14)$$

Also, $\pi_1\{K_{ih} - (\mu_i)\} \approx \pi_1(K_{6ih})$ and therefore the fundamental group $\pi_1(K_{6ih})$ is isomorphic to identity group.

Theorem 2. The fundamental group of any retraction of the geometric Narlikar and Karmarkar space induces the fundamental group of retractions of every pure chaotic Narlikar and Karmarkar space and therefore the fundamental group is isomorphic to identity group.

Proof. Let $r_g : k_{oh} \longrightarrow \bar{k}_{oh}$, k_{oh} is the geometric Narlikar and Karmarkar space and $\bar{k}_{oh} \subset k_{oh}$ then, there are induced retractions: $r_1 : k_{1h} \longrightarrow \bar{k}_{1h}$, $\bar{k}_{1h} \subset k_{1h}$, $r_2 : k_{2h} \longrightarrow \bar{k}_{2h}, \bar{k}_{2h} \subset k_{2h}, \ldots, r_\infty : k_{\infty h} \longrightarrow \bar{k}_{\infty h}$, $k_{\infty h} \subset \bar{k}_{\infty h}$. If $\bar{r}_m : k_{oh} \longrightarrow \bar{P}_{oh}$, then, $\bar{r}_1 : k_{1h} \longrightarrow \bar{P}_{1h}$, $\bar{r}_2 : k_{2h} \longrightarrow \bar{P}_{2h}, \ldots, r_\infty : k_{\infty h} \longrightarrow \bar{k}_{\infty h}$.

Where $\bar{r}_m$ is the minimum retraction of the geometric Narlikar and Karmarkar space and therefore the fundamental group is isomorphic to identity group. Also, this retraction imply to minimum retraction of pure chaotic Narlikar and Karmarkar space. The chaotic retraction is homeomorphic to the geometric retraction of the chaotic Narlikar and Karmarkar space but the chaotic Narlikar and Karmarkar space is not homeomorphic to its retraction again therefore the fundamental group is isomorphic to identity group .

Theorem 3.The fundamental group of the retraction of pure chaotic Narlikar and Karmarkar space does not induce the fundamental group of a retraction of geometric Narlikar and Karmarkar space. Also, the fundamental group is isomorphic to identity group.

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Proof. Let $r_{1h} : k_{123\ldots\infty h} \longrightarrow \bar{k}_{123\ldots\infty h}$, be a pure chaotic retraction of $k_{123\ldots\infty h}$ into $\bar{k}_{123\ldots\infty h}$, then there is an induced sequence of retractions.

$$k_{1h} \xrightarrow{r_{1h}} \bar{k}_{1h} \xrightarrow{r_{2h}} \bar{k}_{1h} \longrightarrow \ldots \xrightarrow{r_{\infty h}} \bar{k}_{1h}$$

$$k_{2h} \xrightarrow{r_{2h}} \bar{k}_{2h} \xrightarrow{r_{2h}} \bar{k}_{2h} \longrightarrow \ldots \xrightarrow{r_{\infty h}} \bar{k}_{2h}$$

$$k_{\infty h} \xrightarrow{r_{1h}} \bar{k}_{\infty h} \xrightarrow{r_{2h}} \bar{k}_{\infty h} \ldots \xrightarrow{r_{\infty h}} \bar{k}_{\infty h},$$

Also, $k_{1h} \xrightarrow{r_{1h}} k_{oh} \xrightarrow{r_{2h}} k_{oh} \longrightarrow \ldots \xrightarrow{r_{\infty h}} k_{oh}$. This means that all r_{ih} on k_{oh} is the identity map. In this case, the fundamental group is isomorphic to identity group .

Theorem 4. The fundamental groups of the end of the limits of the foldings of chaotic n-dimensional Narlikar and Karmarkar space is isomorphic to identity group.

Proof. Let $f_1 : k^n_{ih} \to k^n_{ih}$, $f_2 : f_1(k^n_{ih}) \to f_1(k^n_{ih})$, $f_3 : f_2(f_1(k^n_{ih})) \to f_2(f_1(k^n_{ih}),\ldots,$

$f_n : f_{n-1}(f_{n-2}\ldots f_1(k^n_{ih})\ldots) \longrightarrow f_{n-1}(f_{n-2}\ldots f_1(k^n_{ih})\ldots)$, then $\lim\limits_{n\to\infty} f_n(f_{n-1}(f_{n-2}\ldots f_1(k^n_{ih})\ldots) = k^{n-1}_{oh}$, which is the chaotic Narlikar and Karmarkar space of dimension $n-1$. Also, if we consider

$F_1 : k^{n-1}_{ih} \longrightarrow k^{n-1}_{ih}$, $F_2 : F_1(k^{n-1}_{ih}) \longrightarrow F_1(k^{n-1}_{ih}),\ldots$ $F_3 : F_2(F_1(k^{n-1}_{ih}) \longrightarrow F_2(F_1(k^{n-1}_{ih}),\ldots,$
$F_m : F_{m-1}(F_{m-2}\ldots F_1(k^{n-1}_{ih})\ldots) \longrightarrow F_{m-1}(F_{m-2}\ldots F_1(k^{n-1}_{ih})\ldots)$, then

$\lim\limits_{m\to\infty} f_m(f_{m-1}(f_{m-2}\ldots f_1(k^{n-1}_{ih})\ldots) = k^{n-2}_{ih}$, which is the chaotic Narlikar and Karmarkar space of dimension $(n-2)$ Consequently, $\lim\limits_{S\to\infty}\lim\limits_{m\to\infty}\lim\limits_{n\to\infty}\ldots H_5(F_m(Fg_1(k^n_{ih})\ldots) =$, zero dimensional chaotic Narlikar and Karmarkar space, and therefore the fundamental group is isomorphic to identity group .

Corollary 1. The fundamental groups of the end of the limits of foldings of chaotic Narlikar and Karmarkar space is equivalent to the fundamental groups of the end of retractions.

Theorem 5. The fundamental groups of the fractal retraction of the geometric chaotic Narlikar and Karmarkar space induces the fundamental groups of a fractal dimension of the pure chaotic Narlikar and Karmarkar space but the inverse is not true.

Proof. Let k_{ih} be a chaotic Narlikar and Karmarkar space such that k_{oh} is the geometric of chaotic Narlikar and Karmarkar space and let any point $p = (x_1, x_2, \ldots, x_{n+1}) \in k_{oh}$. If $\forall p_i \in k_{oh}$, $p_i = (x_1, x_2, \ldots, \varepsilon\ x_{n+1})$, there are chaotic points : $P_1 = (x_1, x_2, \ldots., \varepsilon_1\ x_{n+1})$
$P_2 = (y_1, y_2, \ldots., \varepsilon_2\ y_{n+1}),\ldots,$ $P_n = (z_1, z_2, \ldots., \varepsilon_n\ z_{n+1}), \varepsilon_j \square\ 1, \varepsilon_j \to 0,$ $j = 1, 2, \ldots n$ then the dimension of $k_{ih} = n + \dfrac{1}{p}$, p is a positive integer $p>1$ (i.e $k_{ih}^{n+\frac{1}{p}}$). Then, the dimension of

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k_{1h}, k_{2h}, ..., and $k_{\infty h}$ is $n+\frac{1}{p}$. If any point on k_{oh} is in the form $P_{oh}=\left(\varepsilon_1 x_1,\varepsilon_2 x_2,...,\varepsilon_{n+1}x_{n+1}\right)$ and $\varepsilon_j \square 1$, where $\varepsilon_j=\max\,\varepsilon_1$ then $k_{oh}^{\frac{1}{p}}$,p>1, and any point on k_{1h} is $p_{1h}=\left(\varepsilon_{1h}x_{1h},\varepsilon_{2h}x_{2h},...,\varepsilon_{(n+1)h}x_{(n+1)h}\right),\varepsilon_{ij}\square 1$, where $\varepsilon_{jh}=\max\,\varepsilon_{ih}$. Then, the fractal retraction of the chaotic Narlikar and Karmarkar space $r_i:\left(x_1,x_2,....,x_{n+1}\right)\to\left(x_1,x_2,....,\varepsilon_{1x_i},...,x_{n+1}\right)$ is the retraction of the coordinate x_i where $\varepsilon_1 \square 1$ there are many retraction $r_{i(jh)}$ such that $r_{i(jh)}:(x_1^j,x_2^j,...,x_{n+1}^j)\to(x_1^j,...,\varepsilon_1 x_i^j,x_{n+1}^j)$ and this retraction induces a fractal dimension $k_{ih}^{n+\frac{1}{p}}$ $\forall i=1,2,3,...$ Therefore, the fractal retraction of the geometric chaotic Narlikar and Karmarkar space is the retraction of the coordinate, it induces a retraction of every pure chaotic Narlikar and Karmarkar space and this retractions will be also in dimension but the inverse is not true.

Theorem 6. The fundamental groups of The fractal folding of the chaotic Narlikar and Karmarkar space with fractal dimension is the same as the fundamental groups of a chaotic Narlikar and Karmarkar space with dimension n.

Proof . Let $f:\left(x_1,x_2,....,x_n\right)\to\left(x_1,x_2,....,\alpha_n\right)$, $\alpha<<1$. Then the fractal folding of chaotic Narlikar and Karmarkar space the fractal dimension

$f_{oh}: k_{oh}^{n+\frac{1}{p}}\to k_{oh}^{n+\frac{1}{p}}$, where

$$f_{oh}:\left(x_1,x_2,....,x_n,\varepsilon_1 x_{n+1}\right)\to\left(x_1,x_2,....,x_n,\varepsilon_1\left|x_{n+1}\right|\right)\ ,\ \left|\varepsilon_1\right|<1$$

$f_{1h}: k_{1h}^{n+\frac{1}{p}}\to k_{1h}^{n+\frac{1}{p}}$, where $f_{1h}:\left(x_1,x_2,....,x_n,\varepsilon_1 x_{n+1}\right)\to\left(x_1,x_2,....,x_n,\varepsilon_2 x_{n+1}\right)$ where $\varepsilon_2<\varepsilon_1$,

$f_{2h}:\left(x_1,x_2,....,x_n,\varepsilon_2 x_{n+1}\right)\to\left(x_1,x_2,....,x_n,\varepsilon_3 x_{n+1}\right)$ where $\varepsilon_3<\varepsilon_2$,

$f_{nh}:\left(x_1,x_2,....,x_n,\varepsilon_n x_{n+1}\right)\to\left(x_1,x_2,....,x_n,\varepsilon_{n+1}x_{n+1}\right)$ where $\varepsilon_{n+1}<\varepsilon_n$, $\varepsilon_{n+1}\to 0$

$\lim\limits_{n\to\infty} f_{nh}\left(k_{oh}^{\frac{1}{n+p}}\right)=k_{oh}^{n}$ and therefore the fundamental group is isomorphic to identity group . Now, there are corresponding induced sequences of folding

$F_{ih}: k_{1h}^{n+\frac{1}{p}}\to k_{1h}^{n+\frac{1}{p}}$, $F_{2h}: k_{2h}^{n+\frac{1}{p}}\to k_{2h}^{n+\frac{1}{p}}$,..., $F_{\infty h}: k_{\infty h}^{n+\frac{1}{p}}\to k_{\infty h}^{n+\frac{1}{p}}$, and F_{1h1} , F_{1h2},..., $F_{1hm}, F_{2h1}, F_{2h2},..., F_{2hm}, F_{3h1}, F_{3h2},..., F_{3hm},..., F_{nh1}, F_{nh2},..., F_{nhm}$ such that

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$$\lim_{m\to\infty} f_{1\,h\,m}\left(k_{1\,h}^{\frac{1}{n+p}}\right)=k_{1\,h}^{n},\ \lim_{m\to\infty} f_{2\,h\,m}\left(k_{2\,h}^{\frac{1}{n+p}}\right)=k_{2\,h}^{n},\ldots,\ \lim_{m\to\infty} f_{n\,h\,m}\left(k_{\infty\,h}^{\frac{1}{n+p}}\right)=k_{\infty\,h}^{n}.$$

and also the fundamental group is isomorphic to identity which is the required result.

Conclusion

The present work deals with what we consider to be the fundamental group of fractal retraction of chaotic Narlikar and Karmarkar space. The fundamental groups of the geodesics of chaotic Narlikar and Karmarkar space are deduced. The fundamental groups of the folding of chaotic Narlikar and Karmarkar space are presented.

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